

# Non-Commutative Rings and their Applications VIII

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## Direct Complements Almost Unique

**Abstract:** Direct complements in a right  $R$ -module  $M$  are said to be almost unique if, whenever  $M = A \oplus B = A \oplus C$ , then  $(B + C)/B \ll M/B$  (also  $(B + C)/C \ll M/C$ , by symmetry). We will show that this new class of modules lies strictly between the dual-square-free and the summand-dual-square-free modules. While it is an open question whether right quasi-duo (equivalently, right dual-square-free) rings are left-right symmetric, we will prove that both notions “summand-dual-square-free” and “direct complements almost unique” are left-right symmetric. Furthermore, we will show that direct complements in a module  $M$  are almost unique iff idempotents in  $S := \text{End}(M_R)$  are central modulo  $\nabla(M)$ , where  $\nabla(M) := \{f \in S : \text{Im} f \ll M\}$ . As an immediate consequence, if  $M$  is an epi-projective module, then direct complements in  $M$  are almost unique iff  $M$  is strongly perspective (i.e. if  $A$  and  $B$  are isomorphic direct summands of  $M$  and  $M = A \oplus X$ , then  $M = B \oplus X$ ). In particular, direct complements in a ring  $R$  are almost unique iff  $R$  is strongly perspective. Moreover, if  $M$  is a module with the finite exchange whose direct complements are almost unique, then  $M$  is clean, strongly perspective, and satisfy the full exchange.

This is a joint work with Yasser Ibrahim of both Taibah and Cairo Universities.